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Numerical Analysis

B.Sc - III (H) PAPER - VIII

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Transcendental Equation The equations which involve transcendental functions like $\sin x$, $\cos x$, $\log x$, e^x etc are called transcendental equations.

Fundamental theorem If $f(x)$ is continuous from $x = a$ to $x = b$ and if $f(a)$ and $f(b)$ have opposite signs then there is at least one real root between a and b.

To show that bisection method always converges.

Let $[a_n, b_n]$ be the interval at the nth step of bisection containing the root of the equation $f(x) = 0$. Let x_n be the nth approximation of the root. Initially $a_1 = a$ and $b_1 = b$ and $x_1 = \frac{a+b}{2}$ i.e.

Thus the first approximation of the root is $\frac{a+b}{2}$

Thus $a < x_1 < b$

Now either the root lies in $[a, x_1]$ or in $[x_1, b]$. In either case, the length of the sub-interval is $\frac{b-a}{2}$

After 2 bisections, the subinterval containing the root of the equation will be $[a_1, b_1]$ where $[a_1, b_1] \subset [a, b]$ and the length of the subinterval $[a_1, b_1]$ will be $\frac{b-a}{2^2}$

Similarly after n bisections, the sub-interval containing the root of the equation will be $[a_n, b_n]$ where $[a_n, b_n] \subset$

$[a_n, b_n] \subset [a_{n-1}, b_{n-1}]$ and the length of the sub-interval will be $\frac{b-a}{2^n}$

Thus $(a, a_1, a_2, ---)$ is a monotonically increasing sequence bounded by b. Hence the sequence $[a_n]$ tends to a limit, say l.

Similarly $(b, b_1, b_2, ---)$ is a monotonically decreasing sequence bounded below by a. Hence the sequence $[b_n]$ tend to a limit, say m

Moreover $Lt \frac{b-a}{2^n} \to 0$ as $n \to \infty$

Hence both the limits l and m are the same i.e., $l = m$. Thus the sequence $[x_n]$ converges necessarily to a root of the equation $f(x) = 0$

But the convergence is very slow.